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Classical Transport Theory for a Gluon Plasma

Xiangjun Chen,[1] Gang Wang,[1] Weining Zhang,[1] Lei Huo,[1] and Pengfei Zhuang[2]

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The classical picture of a pure gluon plasma is given and classical transport equations for gluons in the system are derived and compared with quantum equations for gluons and classical equations for quarks.

1. INTRODUCTION

It is predicted by lattice simulations [1] that a phase transition from a hadron gas to a quark–gluon plasma (QGP) can occur at high temperature and density. The estimated critical temperature for the deconfinement phase transition is about 150 MeV, which is so high that a QGP is expected to be formed in laboratories only through relativistic heavy-ion collisions. However, because of the very short lifetime of the collision zone, the highly excited quark–gluon system may spend a considerable fraction of its life in a nonthermalized, preequilibrium state. The dynamical tool to treat dissipative processes in heavy ion collisions and the approach to local thermal equilibrium is in principle nonequilibrium quantum transport theory. A relativistic and quantum kinetic theory [2] for quarks and gluons has been derived based on covariant Wigner operators. Some preliminary applications [3] to the QGP, such as linear color response, color correlations, and collective plasma oscillations, have been discussed by using a semiclassical expansion [4] of quantum transport theory. For the classical transport theory of QGP, there have been some recent applications, for example, the classical kinetic theory can lead to the "HTL" of QCD [5] and has been developed into a nonperturbative method for not QCD [6]. In classical transport theory, where the quarks move in a classical gluon field, the classical transport equation for quarks was

[1] Physics Department, Harbiin Institute of Technology, Harbin 150001, China.
[2] Physics Department, Tsinghua University, Beijing 100084, China.

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investigated by Heinz [7]. But the classical transport equation for gluons as classical particles as well as the classical transport theory for gluons are rarely discussed because gluons are massless particles. In order to set up classical transport theory for of gluons, in this paper we only consider a pure gluon plasma system for simplicity. In Section 2 we give a classical picture of the gluon plasma. In Section 3 we study the classical motion equations of gluons. In Section 4 we directly derive the classical transport equations for gluons from the equations of motion and discuss the relation between the classical transport equations for gluons and for quarks. In Section 5 we discuss the relation between the classical and the quantum transport equations for gluons.

2. CLASSICAL PICTURE FOR A GLUON PLASMA

In order to develop a classical transport theory of gluons, we consider a pure gluon plasma system and first give a classical picture of the system. In the classical model of a quark–gluon plasma, a quark is thought of as a classical relativistic particle moving in a gluon field. The gluon field includes the external field and the self-consistent field which is excited by the collective motion of other quarks in the plasma. A gluon is a particle with color charge and similar to a quark in being point-like. Therefore in the classical picture of the gluon plasma, a gluon is treated as a relativistic colored particle with covariant coordinate x^μ, momenta $p^\mu = \dot{x}^\mu$ ($\dot{x}^\mu = dx^\mu/d\tau$, τ is the proper time), and color charges Q^a. For simplicity, the gluon spin is neglected here. The system is described by the one-particle distribution function $f(x, p, Q)$ for gluons, which denotes the probability to find a classical gluon at a given point (x, p, Q) in phase space.

There are interactions among the gluons since the gluon is a colored particle. In QCD, there are three-gluon and four-gluon interaction vertices. In the classical picture of a gluon plasma, the three-gluon interaction and the four-gluon interaction remain. The plasma is produced at high temperature, and the collisions between gluons are very fast. In transport theory, three-body collision is generally ignored. Hence we do not consider three-gluon collision here. There are three-gluons interactions, where a two-gluon collision can produce one gluon so that the number of gluons is not conserved in the collision. This differs from the size for quarks because an elastic two-quark collision cannot produce one quark. In the process of two-gluon collision, gluon momentum and energy are conserved. Hence the momentum and energy of the system are conserved due to collisions of gluons.

Using the one-particle distribution function, we can get an ensemble average value of physical quantities. The gluon number current is defined by

$$n^{\mu} = \int p^{\mu} f(x, p, Q)\, dP\, dQ \tag{1}$$

where $dP = 2\theta(p_0)\, \delta(p^2)\, d^3p/(2\pi)^3$, $dQ = \delta(Q^a Q_a - q^2)\, \delta(d_{abc} Q^a Q^b Q^c - \bar{q}^3)\, d^8Q$. Here $n^0 = \int f\, d^3p/(2\pi)^3\, dQ$ is the number density of gluons. The color current is defined by

$$j_a^{\mu} = \int Q^a p^{\mu} f(x, p, Q)\, dP\, dQ \tag{2}$$

The color current can excite a self-consistent field which satisfies the color Maxwell equation

$$\partial_{\mu} F_a^{\mu\nu} = j_a^{\nu} \tag{3}$$

The energy-momentum tensor is defined as

$$T^{\mu\nu} = \int p^{\mu} p^{\nu} f(x, p, Q)\, dP\, dQ \tag{4}$$

In the above equation, $T^{00} = \int p^0 f\, d^3p\, dQ/(2\pi)^3$ is the energy density of the gluon plasma, and $P(x) = T^{ij}\, \delta_{ij} = \int pf\, d^3p\, dQ/(2\pi)^3$ is the pressure of gluons in the direction of motion.

3. CLASSICAL EQUATIONS OF MOTION FOR GLUONS

In the section, we study the classical equations of motion for gluons. In a gluon plasma, gluons interact with other gluons in a very complex manner. For simplicity, we use the mean-field method and consider the interactions in terms of gluons interacting with a self-consistent field excited by other gluons in the plasma, or the gluon moves in an excited gauge field. With the same method as used for dealing with quarks in ref. 8, the equations of motion for gluons can be derived from the classical limit of relativistic quantum mechanics. As proposed by Fock and Schwinger [9], the invariant quantum equations of motion are determined by the Heisenberg equation in the proper-time representation,

$$\dot{A} = \frac{dA}{d\tau} = i[H, A] \tag{5}$$

where H is the Hamiltonian of the system. In relativistic quantum mechanics, a gluon satisfies the homogeneous Yang–Mills equations in the color chargeless region,

$$\begin{gathered} D^{\mu} F_{\mu\nu} = 0 \\ D_{\mu} F_{\nu\lambda} + D_{\lambda} F_{\mu\nu} + D_{\nu} F_{\lambda\mu} = 0 \end{gathered} \tag{6}$$

where $F^{\mu\nu} = -F_a^{\mu\nu}Q^a$ is the field tensor, $Q^a = -\lambda^a/2$ the color charge, and $D_\mu = \partial_\mu - igA_\mu$ the covariant derivative. With the definitions $F_{0i} = E_i$ and $F_{ij} = \epsilon_{jik}B_k$, Eqs. (6) can be rewritten as

$$D^0E_i + D^j\epsilon_{jik}B_k = 0$$
$$D_0\epsilon_{ijk}B_k + D_jE_i - D_iE_j = 0 \tag{7}$$

We introduce a two-component vector and γ matrices

$$\psi = \begin{pmatrix} \vec{E} \\ i\vec{B} \end{pmatrix}, \qquad (S_i)_{jk} = (-i\epsilon_i)_{jk}, \qquad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{S} \\ -\vec{S} & 0 \end{pmatrix} \tag{8}$$

The Yang–Mills equations are now compactly written as a Dirac-like equation

$$i\gamma_\mu D^\mu\psi = 0 \tag{9}$$

Unlike Abelian fields, which satisfy the Maxwell equation $\partial_\mu F^{\mu\nu} = 0$, the covariant derivative in Eq. (9) reflects the non-Abelian character of gluons. In this equation, ψ represents the gluon under consideration and A_μ in the covariant derivative D_μ denotes the self-consistent field excited by other gluons.

Therefore, according to the proposal by Nambu [10], the Hamiltonian describing the evolution of gluons in Eq. (5) can be taken as

$$H = i\gamma_\mu D^\mu \tag{10}$$

By substituting the Hamiltonian (10) into Eq. (5), we obtain the classical equations of motion

$$\dot{p}_\mu = g\dot{x}^\nu F_{\nu\mu} = gF^a_{\mu\nu}p^\nu Q_a$$
$$\dot{Q}_a = -gf_{abc}p^\mu A^b_\mu Q_c \tag{11}$$

The equations of motion are the are same as the classical equations of motion for quarks [2] when the spin of the quarks is neglected. Why are they same? In the classical transport theory of QGP, a quark moves in a gluon field. The interaction vertex between a quark and a gluon is the interaction of a vector and a color current. In the gluon plasma, when the Yang–Mills equations are written as a Dirac-like equation, the vertex of the gluon and the self-consistent field is similar to the case of quarks. Since the interactions have the same form, the motion has the same character. In QCD, the three-gluon interaction is proportional to the coupling constant g and the four-gluon interaction is proportional to g^2. So the probability to produce one gluon in a two-gluons collision is larger than that to produce two gluons in the perturbative region. In the classical theory of the gluon plasma the case is the same because in the Dirac-like equation the interaction of a vector and a color current is the classical analogue of the three-gluon interaction, not including the four-gluon interaction.

4. CLASSICAL TRANSPORT EQUATION FOR GLUONS

The gluon plasma exists in a nonequibrium state generally, and the distribution function evolves with space-time. In a relativistic kinetic theory,

$$\frac{df}{d\tau} = \dot{x}^\mu \frac{\partial f}{\partial x^\mu} + \dot{p}^\mu \frac{\partial f}{\partial p^\mu} + \dot{Q}^a \frac{\partial f}{\partial Q^a} = C(x, p, Q) \tag{12}$$

where τ is the proper time, $C(x, p, Q)$ is the collision term describing short-range two-body collisions, and $\dot{p}$ and $\dot{Q}$ satisfy the classical equations of motion for the gluons of the last section. Then the classical transport equation for the gluons is

$$\begin{aligned} p^\mu \partial_\mu f(x, p, Q) &= g p^\mu F^a_{\mu\nu} Q^a \, \partial_p^\nu f(x, p, Q) \\ &\quad + g f_{abc} p^\mu A^b_\mu Q_c \, \partial_Q^a f(x, p, Q) + C(x, p, Q) \end{aligned} \tag{13}$$

The symbols ∂_p^ν and ∂_Q^a indicate the momentum and charge derivatives $\partial/\partial p_\nu$ and $\partial/\partial Q^a$, respectively.

Both gluons and quarks carry color charges; what are the transport properties between them? The classical transport equation for quarks is [8]

$$\begin{aligned} & p^\mu \, \partial_\mu f(x, p, Q, S) \\ &= \left[g Q^a p^\mu F^a_{\mu\nu} - \frac{g}{2} (D_\nu S^{\alpha\beta} F_{\alpha\beta})^a Q^a \right] \partial_p^\nu f(x, p, Q, S) \\ &\quad - \left[g Q_a (F^a_{\mu\lambda} S^\lambda + \frac{1}{m^3} (p_\mu S^\nu - p^\nu S_\mu)(D_\nu \overline{F}_{\alpha\beta})^a p^\alpha S^\beta) \right] \partial_S^\mu f(x, p, Q, S) \\ &\quad + \left[g f_{abc} (p^\mu A^b_\mu + \frac{1}{2} S^{\alpha\beta} F^b_{\alpha\beta}) Q_c \right] \partial_Q^a f(x, p, Q, S) + C(x, p, Q, S) \end{aligned} \tag{14}$$

When the spin effect of quarks is ignored, Eq. (14) is greatly simplified as

$$\begin{aligned} p^\mu \partial_\mu f(x, p, Q) &= g p^\mu F^a_{\mu\nu} Q^a \, \partial_p^\nu \dot{f}(x, p, Q) \\ &\quad + g f_{abc} p^\mu A^b_\mu Q_c \, \partial_Q^a f(x, p, Q) + C(x, p, Q) \end{aligned} \tag{15}$$

which is the same as the classical transport equation (13) for the gluon distribution function. Therefore Eq. (15) can be regarded as a general transport equation for any classical colored particle. In the semiclassical limit of quantum transport theory, one finds that the semiclassical transport equation for gluons has a similar space-time-structure as that for quarks [2] when the spin effect of quarks is neglected even through their Wigner functions are not the same because the Wigner function of a gluon is an 8×8 matrix in the adjoint representation space of the $SU(3)$ group and the Wigner function of a quark is a 3×3 matrix in the fundamental representation space of the $SU(3)$ group. This implies that the classical transport equation for gluons should have the same form as that for quarks. That the classical transport equations for glons and for quarks are the same indicates that the classical theory is consistent with the quantum theory.

In Section 2, we defined the color current j_a^μ and the energy-momentum tensor $T^{\mu\nu}$. Due to the conservation of the color current and of the energy-momentum of the system, we can prove using the classical transport equation for gluons that

$$D_\mu j_a^\mu = 0 \tag{16}$$

$$\partial_\mu T^{\mu\nu} = 0 \tag{17}$$

For the comparison in the next section with the quantum transport equations, we further consider the color moments of the kinetic equation. Defining the color moments

$$\begin{aligned} f^0(x, p) &= \int f(x, p, Q)\, dQ \\ f^a(x, p) &= \int Q^a f(x, p, Q)\, dQ \end{aligned} \tag{18}$$

and integrating Eq. (13) over the color charge Q, we obtain the classical transport equation

$$p^\mu \partial_\mu f^0(x, p) = g p^\mu F^a_{\mu\nu}\, \partial_p^\nu f^a(x, p) + C_0(x, p) \tag{19}$$

for the color-singlet distribution f^0, where $C_0(x, p) = \int C(x, p, Q)\, dQ$ is the zeroth-order moment of the collision term $C(x, p, Q)$. Timing Eq. (13) by Q^a, then integrating it over the color charge Q, we obtain

$$p^\mu \partial_\mu f^a(x, p) = g p^\mu F^b_{\mu\nu}\, \partial_p^\nu f^{ab}(x, p) - g f_{abc} p^\mu A^b_\mu f^c(x, p) + C_a(x, p) \tag{20}$$

for the color-octet distribution f^a, where $C_a(x, p) = \int Q_a C(x, p, Q)\, dQ$ is the first-order moment of the collision term and $f^{ab} = \int Q^a Q^b f(x, p, Q)\, dQ$ is the second-order moment of the distribution function $f(x, p, Q)$.

5. COMPARISON WITH QUANTUM TRANSPORT EQUATIONS FOR GLUONS

For a quantum QGP system the behavior of gluons is described by the gluon Wigner operator. In the general case the quantum kinetic equation is very complicated and needs to be simplified before applications can be made. In mean-field approximation the function $G^{\mu\nu}$, which is the ensemble average of the Wigner operator fluctuation, satisfies the following covariant kinetic equation [2]:

$$
\begin{aligned}
&p^\rho D_\rho G_{\mu\nu}(x, p)\\
&\quad = \frac{g}{2} p^\sigma \partial_p^\tau \int_0^1 ds\ \{[e^{s\Delta}F_{\sigma\tau}, G_{\mu\nu}]_L + [G_{\mu\nu}, e^{-s\Delta}F_{\sigma\tau}]_R\}\\
&\qquad + \frac{ig}{4}\,\partial_p^\tau \int_0^1 ds\ s\{[e^{s\Delta}F_{\tau\sigma}, D^\sigma G_{\mu\nu}]_L - [D^\sigma G_{\mu\nu}, e^{-s\Delta}F_{\tau\sigma}]_R\}\\
&\qquad + \frac{ig^2}{8}\,\partial_p^\sigma\partial_p^\tau \int_0^1 ds\ s \int_0^1 d\bar{s}\ \{[e^{s\Delta}F_{\sigma\eta}, [e^{\bar{s}\Delta}F^\eta_\tau, G_{\mu\nu}]_L + [G_{\mu\nu}, e^{-\bar{s}\Delta}F^\eta_\tau]_R]_L\\
&\qquad - [[e^{\bar{s}\Delta}F^\eta_\tau, G_{\mu\nu}]_L + [G_{\mu\nu}, e^{-\bar{s}\Delta}F^\eta_\tau]_R, e^{-s\Delta}F_{\sigma\eta}]_R\}\\
&\qquad + g\{[e^{\Delta}F_{\mu\lambda}, G^\lambda_\nu]_L - [G_{\mu\lambda}, e^{-\Delta}F^\lambda_\nu]_R\}
\end{aligned}
\tag{21}
$$

where s, $\bar{s}$ are the variables of the integral and $[O, A \otimes B]_L \equiv [O, A] \otimes B$ and $[A \otimes B, O]_R, \equiv A \otimes [B, O]$ are left and right commutators, respectively. Since $G_{\mu\nu}$ is a matrix in color space, due to the relation between the color charges and the Gell-Mann matrices, $Q_a = -\frac{1}{2}\lambda_a$, there exist in Eq. (21) commutators and anticommutators defined in color space. The triangle operator Δ is defined as $\Delta = (i/2)D_\mu\ \partial_p^\mu$, where the momentum derivative ∂_p^μ acts only on the function $G_{\mu\nu}$. In order to better understand the structure of the kinetic equation, we consider its expansion in the triangle operator Δ. Note that since each triangle operator is accompanied by an a $\hbar$, the Δ expansion is equivalent to a semiclassical expansion in $\hbar$. To first order in Δ and keeping only first-order derivatives of $G_{\mu\nu}$, the mean-field kinetic equation (21) becomes

$$
\begin{aligned}
&p^\rho D_\rho G_{\mu\nu}(x, p)\\
&\quad = \frac{g}{2} p^\sigma \partial_p^\tau \{[F_{\sigma\tau}, G_{\mu\nu}]_L + [G_{\mu\nu}, F_{\sigma\tau}]_R\} + g\{[F_{\mu\lambda}, G^\lambda_\nu]_L - [G_{\mu\lambda}, F^\lambda_\nu]_R\}\\
&\qquad + \frac{ig}{2}\,\{[D_\sigma F_{\mu\lambda}, \partial_p^\sigma G^\lambda_\nu]_L + [\partial_p^\sigma G_{\mu\lambda}, D_\sigma F^\lambda_\nu]_R\}
\end{aligned}
\tag{22}
$$

In order to compare with the classical transport equations derived in the last section, we insert the decomposition of $G_{\mu\nu}$ and $F_{\mu\nu}$ in color space,

$$G_{\mu\nu}(x, p) = G^{ab}_{\mu\nu} \frac{\lambda_a}{2} \otimes \frac{\lambda_b}{2}$$

$$F_{\mu\nu}(x) = F^{a}_{\mu\nu} \frac{\lambda_a}{2} \tag{23}$$

into Eq. (22); then the color component $G^{ab}_{\mu\nu}$ satisfies the kinetic equation

$$\begin{aligned}
&p^\rho \partial_p G^{ab}_{\mu\nu} - ip^\rho A^c_\rho(-if^{cad})G^{db}_{\mu\nu} + iG^{ad}_{\mu\nu}p^\rho A^c_\rho(-if^{cdb}) \\
&= \frac{g}{2} p^\sigma\ \partial^\tau_p F^d_{\sigma\tau}[(-if^{dac})G^{cb}_{\mu\nu} \\
&\quad + G^{ac}_{\mu\nu}(-if^{dcb})] + g[F^c_{\mu\lambda}(-if^{cad})(G^\lambda_\nu)^{db} - (G_{\mu\lambda})^{ad}F^{\lambda c}_\nu(-if^{cdb})] \\
&\quad + \frac{ig}{2}[\partial_\sigma F^c_{\mu\lambda}(-if^{cad})(\partial^\sigma_p G^\lambda_\nu)^{db} + f^{ehc}A^e_\sigma F^h_{\mu\lambda}(-if_{cad})(\partial^\sigma_p G^\lambda_\nu)^{db} \\
&\quad + (\partial^\sigma_p G^{ad}_{\mu\lambda})\ \partial_\sigma F^{\lambda c}_\nu(-if^{cdb}) + (\partial^\sigma_p G^{ad}_{\mu\lambda}) f^{ehc}A^e_\sigma(F^\lambda_\nu)^h(-if^{cdb})]
\end{aligned} \tag{24}$$

With the definitions $(F^a)^{bc} = (-if^a)^{bc}$, $F_{\sigma\tau} = F^a_{\sigma\tau}F^a$, $D_\rho = \partial_\rho - iA^c_\rho F^c$, and $D_\rho Q = \partial_\rho Q - i[A_\rho, Q]$, the above equation can be compactly written as

$$\begin{aligned}
p^\rho D_\rho G_{\mu\nu} &= \frac{g}{2} p^\sigma\ \partial^\tau_p\{F_{\sigma\tau}, G_{\mu\nu}\} + g(F_{\mu\lambda}G^\lambda_\nu - G_{\mu\lambda}F^\lambda_\nu) \\
&\quad + \frac{ig}{2}[(D_\sigma F_{\mu\lambda})\ \partial^\sigma_p G^\lambda_\nu + \partial^\sigma_p G_{\mu\lambda}(D_\sigma F^\lambda_\nu)]
\end{aligned} \tag{25}$$

for the 8×8 matrix $G_{\mu\nu} = \{G^{ab}_{\mu\nu}\}$ in the adjoint representation of the $SU_c(3)$ group. Since $G_{\mu\nu}$ is reducible, it can be decomposed in the following way:

$$G_{\mu\nu} = G^0_{\mu\nu}\frac{I}{8} + G^a_{\mu\nu}\frac{F^a}{3} + \cdots \tag{26}$$

where the one-dimensional and eight-dimensional representations $G^0_{\mu\nu}$ and $G^a_{\mu\nu}$ correspond to the color singlet and color octet, respectively. Performing the trace in color space for the kinetic equation (25) multiplied by $g_{\mu\nu}$, we obtain the kinetic equation

$$p^\rho\ \partial_\rho G^0 = gp^\sigma F^a_{\sigma\tau}\ \partial^\tau_p G^a + ig(D_\sigma F_{\mu\lambda})^\alpha\ \partial^\sigma_p G^{\lambda\mu}_a \tag{27}$$

for the color singlet $G^0 = Tr(g^{\mu\nu}G_{\mu\nu})$, which is coupled to the color octet $G^a = Tr(g^{\mu\nu}G_{\mu\nu}F^a)$ and $G^{\lambda\mu}_a = Tr(g^{\mu\nu}G^\lambda_\nu F^a)$. As is well known, an important aspect of the covariant approach to transport theory is that the complex kinetic equation can be split into a transport and a constraint equation, where the former is a covariant generalization of the Vlasov–Boltzmann equation and the latter is a quantum extension of the classical mass-shell condition. In the current case the complex kinetic equation (27) is separated into the transport equation

$$p^{\rho}\partial_{\rho}G^{0} = gp^{\sigma}F^{a}_{\sigma\tau}\,\partial_{p}^{\tau}G^{a} \tag{28}$$

and the constraint equation

$$(D_{\sigma}F_{\mu\lambda})^{a}\,\partial_{p}^{\sigma}G_{a}^{\lambda\mu} = 0 \tag{29}$$

for the color singlet: Using a similar calculation for the kinetic equation (25) multiplied by $g^{\mu\nu}F^{a}$, we obtain a complex kinetic equation for the color octet which consists of a real part

$$p^{\rho}\,\partial_{\rho}G^{a} = gp^{\sigma}F^{b}_{\sigma\tau}\,\partial_{p}^{\tau}G^{ab} - gf_{abc}p^{\mu}A^{b}_{\mu}G^{c} \tag{30}$$

and an imaginary part

$$f_{abc}F^{b}_{\mu\lambda}G_{c}^{\lambda\mu} + \frac{3}{8}\,(D_{\sigma}F_{\mu\lambda})^{a}\,\partial_{p}^{\sigma}G_{0}^{\lambda\mu} = 0 \tag{31}$$

where G^{ab} is given by the definition $G^{ab} = Tr(g^{\mu\nu}G_{\mu\nu}\{F^{a}, F^{b}/2) = \frac{3}{8}G^{0}\,\delta^{ab}$. It is easy to see that the transport equations derived here by considering the classical limit of the quantum kinetic equation are the same as those obtained directly from the classical equations of motion in the last section, except for the collision terms, which are associated with the higher order derivative terms. The significant difference is that the quantum kinetic equation determines not only the transport properties by the real part, but also the spectral functions by the imaginary part, while the approach discussed in the last section gives only the transport equations.

6. SUMMARY

We have given a classical picture of a pure gluon plasma, and have studied the classical equations of motion and the transport equation for gluons. There are two ways to set up transport equation for a classical system. One is directly from the classical equations of motion, the other is from the classical limit of the corresponding quantum kinetic equations. We have investigated the relation between these two approaches for the gluons in a quark–gluon plasma. From our derivation the two ways approaches are for the transport equations, but the quantum theory can give an extra constraint which determines the spectral function of the particles. We have also shown that at the classical level the quarks and gluons satisfy the same transport equation.

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